Covering a number of important subjects in quantum optics, this textbook is an excellent introduction for advanced undergraduate and beginning graduate students, familiarizing readers with the basic concepts and formalism as well as the most recent advances.

The first part of the textbook covers the semi-classical approach where matter is quantized, but light is not. It describes significant phenomena in quantum optics, including the principles of lasers. The second part is devoted to the full quantum description of light and its interaction with matter, covering topics such as spontaneous emission, and classical and non-classical states of light. An overview of photon entanglement and applications to quantum information is also given. In the third part, nonlinear optics and laser cooling of atoms are presented, where the use of both approaches allows for a comprehensive description. Each chapter describes basic concepts in detail, and more specific concepts and phenomena are presented in ‘complements’.

**Gilbert Grynberg** was a CNRS Senior Scientist at the Laboratoire Kastler Brossel at the Université Pierre et Marie Curie Paris 6, and a Professor at the Ecole Polytechnique. He was a pioneer in many domains, including atomic spectroscopy, nonlinear optics and laser-cooled atoms in optical lattices.

**Alain Aspect** is a CNRS Senior Scientist and Professor at the Institut d’Optique and the Ecole Polytechnique. A pioneer of the field of quantum entanglement, his research covers quantum optics, laser cooling of atoms, atom optics, Bose–Einstein condensates, atom lasers and quantum atom optics. He was awarded the 2010 Wolf Prize in Physics.

**Claude Fabre** is a Professor in the Laboratoire Kastler Brossel at the Université Pierre et Marie Curie Paris 6, and a senior member of the Institut Universitaire de France. His fields of research are quantum optics, atomic and laser physics, both experimentally and theoretically.
Introduction to Quantum Optics
From the Semi-classical Approach to Quantized Light

GILBERT GRYNBERG
Ecole Normale Supérieure, Paris
Ecole Polytechnique

ALAIN ASPECT
Institut d’Optique and Ecole Polytechnique,
Palaisean

CLAUDE FABRE
Université Pierre et Marie Curie and Ecole Normale Supérieure, Paris

With a Foreword by Claude Cohen-Tannoudji
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